Module 3: Measurement

POWERS OF TEN

In the decimal system we work with powers of ten.

10^5	10^4	10^3	10^2	10^1	10^0	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10000}$
10^5	10^4	10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
100,000	10,000	1000	100	10	1	0.1	0.01	0.001	0.0001

From the table you can see that $0.1 = \frac{1}{10} = 10^{-1}$ $\qquad 0.01 = \frac{1}{100} = 10^{-2}$

Dividing by 100 is equivalent to multiplying by 10^{-2}

UNITS OF MEASUREMENT AND CONVERSIONS

Metric Units The metric system is an international standard of measurement based on decimals. It is used to measure weight, liquids, and length. The modern metric system, introduced in Australia in 1970, is called the International System of Units (SI).

Quantity	Unit	Symbol
Length (dimensions)	metre	m
Mass (weight)	gram	g
Volume (liquids)	litre	L (or l or ℓ)
Time	second	s

Examples of other units are the Joule (J) , byte (B) and Watt (W).

Variations to the basic unit The basic unit of time, the second, may vary in quantity to include minutes (1 minute = 60 s) or hours, days, months, etc. The other units may vary in quantities also, large and small quantities of these units often have a prefix to make writing quantities more compact. To indicate these variations, a prefix is added to the basic unit. Note the symbols for each prefix.

For example 0.000001g may be written as 1 mcg or 1µg.

Prefix	Symbol	Example	Multiplication Factor	Order of Magnitude
giga	G	GL for gigalitre = 1,000,000,000L	1000,000,000 = 10^9	9
mega	M	ML for megalitre = 1,000,000 L	1,000,000 = 10^6	6
kilo	k	kg for kilogram = 1000 g	1000= 10^3	3
hecto	h		100= 10^2	2
deka	da		10= 10^1	1
Unit	e.g. m ,g, L or s		1= 10^0	0
deci	d	dL for decilitre = 0.1 L	0.1= 10^{-1}	-1
centi	c	cm for centimetre = 0.01 m	0.01= 10^{-2}	-2
milli	m	ms for millisecond = 0.001 s	0.001= 10^{-3}	-3
micro	mc or µ	mcg or µg for microgram = 0.000001 g	0.000001= 10^{-6}	-6
nano	n	nm for nanometre = 0.000000001 m	0.000000001= 10^{-9}	-9

10^{9}	Giga	G
10^{8}		
10^{7}		
10^{6}	Mega	M
10^{5}		
10^{4}		
10^{3}	kilo	k
10^{2}	hecto	h
10^{1}	deka	da
$10^{0} = 1$	unit	
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-4}		
10^{-5}		
10^{-6}	micro	µ
10^{-7}		
10^{-8}		
10^{-9}	nano	n

To convert from one metric unit to another: You need to know common metric equivalents and how to write one quantity of a unit in terms of another.

Some common metric equivalents:
Units of volume
10^3 millilitres = 1 litre
10^1 decilitres = 1 litre
10^3 litres = 1 kilolitre

Units of length / height
10^3 millimetres = 1 metre
10^2 centimetres = 1 metre
10^3 metres = 1 kilometre

Units of mass
10^6 micrograms = 1 gram
10^3 milligrams = 1 gram
10^3 grams = 1 kilogram

WORKED EXAMPLES

- A mass of 1.8 kg is equivalent to how many grams?

Since 1 kg is 10^3 g, rewrite the kg as 10^3 g. This gives us
1.8 kg = 1.8 x 10^3 g = 1.8 x 1000 g = 1800 g.

- How many g is 540 mg?

Recall that 10^3 mg = 1 g, so dividing by 10^3 gives
1 mg = 10^{-3} g, so rewrite the mg as 10^{-3} g.
This gives 540 mg = 540 x 10^{-3} g = 540 x 0.001 g = 0.54 g.

The **order of magnitude of a number** is, intuitively speaking, the number of powers of 10 contained in the number. The order of magnitude of 86,000 is 4 since the highest power of 10 in this number is 10,000 = 10^4. The order of magnitude of 0.0045 is -3 , since the highest power of 10 in this number is 0.001 = 10^{-3}
The diameter of the DNA helix is approximately 2 nm, so is of the order of magnitude of -9 (equivalent to 10^{-9} m). It is the **same order of magnitude** as the thickness of a cell membrane (between 6 and 9 nm).
The **difference** in the **order of magnitude** between two values is a power of 10. For example, the estimated speed of a fast neutron is 10,000,000 m/s (order of magnitude = 7). This is 4 orders of magnitude (10^4) faster than the estimated speed of a ‘thermal’ neutron which is 2000 m/s (order of magnitude = 3). And a centimetre (10^{-2} m) is 5 orders of magnitude smaller than a kilometre (10^3 m).

CONVERSIONS AND ORDER OF MAGNITUDE

Exercise 1.

1. Complete the table with as much information as you can.

Prefix	Symbol	In decimal notation	Order of magnitude
milli			
	μ		
		100	
mega			
		0.1	
			-2
	k		
giga			9

2. Work out the following metric conversions:

(a) 6 km in metres (d) 1.002 g in micrograms (g) 0.725 kg in grams

(b) 3.2 kL in millilitres (e) 8.214 L in decilitres (h) 52 mm in centimetres

(c) 44 mm in metres f) 120 g in kilograms (i) 40000 ng in micrograms

3. The diameter of Jupiter is 142984 km and the diameter of the Sun is 1.39 Gm. Express both diameters in metres. How many orders of magnitude is the Sun's diameter larger than that of Jupiter?

4. Given that 1 m^3 = 1000000 cm^3 , convert

(a) 3.5 m^3 to cm^3 (b) 5240 cm^3 to m^3

SCIENTIFIC NOTATION

Scientists express numbers so as to show their order of magnitude. This way of representing numbers is called **scientific notation**.

The numeral 86000 has 10000 as its highest power of 10 and so could be written as

$$8.6 \times 10{,}000 = 8.6 \times 10^4$$

Thus 86 000 is equivalent to 8.6×10^4 in **scientific notation.** In this form it can easily be seen that the order of magnitude of 86 000 is 4.

WORKED EXAMPLES

$4.72 \times 10^{-2} = 0.0472$ $3.3 \times 10^0 = 3.3$

$8 \times 10^3 = 8000$ $9.111 \times 10^{-5} = 0.00009111$

$1.004 \times 10^1 = 10.04$ $2.506 \times 10^4 = 25060$

Calculators display numbers that are too large for their display in scientific notation.
The number 7.8×10^{16} may be displayed as 7.8 Exp 16 or 7.8 E 16.
With the help of your calculator, scientific numbers can be easily multiplied, divided, added and subtracted.
To enter numbers in scientific notation quickly into a calculator, calculators have a single button,(EXP or $\times 10^x$), which enters [x10] in a single press, ***do not use multiple button!***
In the worked examples below, I will use EXP to denote the scientific notation button, take the time now to find the scientific notation button on *your* calculator now.

WORKED EXAMPLES

Try the following examples on your calculator.

(i) *Multiply* 9.87×10^5 and 2.45×10^6 , the answer is 2.41815×10^{12}

9.87 EXP 5 x 2.45 EXP 6 = 2.41815E+12

(ii) *Divide* 9.87×10^4 by 3.948×10^7 , the answer is 2.5×10^{-3} , using a calculator:

9.87 EXP 4 ÷ 3.948 EXP 7 = 2.5E-03

(iii) Addition and subtraction of numbers in scientific notation can also be done on your calculator. Try $1.2 \times 10^4 + 3.12 \times 10^3$, your calculator may give the answer in decimal, 15120 or scientific notation, 1.512×10^4 .

(iv) In the following example, the addition in the numerator is done first, then the division.
For $\dfrac{(1.2 \times 10^4 + 3.12 \times 10^3)}{4.5 \times 10^2}$ the answer is 3.36×10^1 or 33.6.
Using your calculator, include brackets around the numerator

(1.2 EXP 4 + 3.12 EXP 3) ÷ 4.5 EXP 2 = 3.36E+01

OR

Press '=' after the numerator is typed in and then do the division. That is,

1.2 EXP 4 + 3.12 EXP 3 = ÷ 4.5 EXP 2 = 3.36×10^1 .

Exercise 2.

1. Rewrite the following using scientific notation

(a) (i) 230 = (ii) 46500 = (iii) 0.02 =

(iv) 0.0051 = (v) $15000 \times 3.9 \times 10^8$ =

and convert these to decimal notation.

(b) (i) 6.54×10^3 = (ii) 4.317×10^5 =

(iii) 1.5×10^{-2} = (iv) $912.65 \times 2.8 \times 10^{-6}$ =

2. The diameter of a human hair is 7.1×10^{-5} m. If you placed 200 of them side by side, what width of hair would you have:

(a) in decimal notation in metres (b) in scientific notation in metres (c) in millimetres?

3. Calculate in scientific notation, using your calculator.

(a) $6.4 \times 10^{3} \quad \times \quad 1.2 \times 10^{5}$

(b) $5.1 \times 10^{2} \quad \times \quad 8.9 \times 10^{-1}$

(c) $9.2 \times 10^{6} \quad + \quad 8 \times 10^{5}$

(d) $4.5 \times 10^{-1} \quad + \quad 7.32 \times 10^{-2}$

(e) $\dfrac{7.7 \times 10^{-7} \quad \times \quad 9 \times 10^{14}}{3.0 \times 10^{8}}$

(f) $\dfrac{(1.2 \times 10^{2} \; - \; 2.0 \times 10^{1})}{5.0 \times 10^{12}}$

(g) $3.04 \times 10^{-4} \quad \times \quad 4.5 \times 10^{10}$

(h) $1.6 \times 10^{5} \quad \div \quad 3.2 \times 10^{4}$

(i) $5.7 \times 10^{5} \quad - \quad 1.2 \times 10^{3}$

(j) $8.84 \times 10^{10} \quad - \quad 6.01 \times 10^{9}$

(k) $\dfrac{2.6 \times 10^{6} \quad \times \quad 1 \times 10^{-9}}{1.3 \times 10^{-5}}$

(l) $\dfrac{(5.7 \times 10^{3} \; + \; 8.2 \times 10^{2})}{8.0 \times 10^{-10}}$

4. The surface areas of the moon is 3.79×10^{7} square kilometres. Only about 41% of the Moon's surface is ever visible from the Earth. Approximately how much of the moon's surface is visible from the Earth, in square kilometres?

SIGNIFICANT FIGURES

The number of **significant figures** tells us the precision of a measurement. For example, say a wooden block is 3.7 cm in width and we use a ruler with only centimetre gradations to measure the width of the block. We can see the width of the block is between 3cm and 4cm, but there are no millimetre (mm) gradations, so we estimate the number of millimetres. We may record 3.6 (or 3.7 or 3.8) cm, the last digit is an estimate. Measurement accuracy depends the smallest gradation of the measuring instrument being used. Significant figures consist of all the certain numbers, in this case '3cm' and one best estimate, which in this case is a tenth of a centimetre or in other words a millimetre.

Non-zero digits always count as significant figures (s.f.).

Captive zeros always count – 2005 has 4 s.f.

Trailing zeros count only if there is a decimal point in the number - 2000 has 1 s.f. but 2000. has 4 s.f. and 2000.0 has 5 s.f.

Leading zeros do not count.

For clarity use scientific notation

Examples The significant figures are underlined in the following examples

$\underline{356}$ has 3 s.f. $\underline{3056}$ has 4 s.f. $0.\underline{356}$ has 3 s.f. $0.00\underline{356}$ has 3 s.f.

$0.\underline{3560}$ has 4 s.f. $0.00\underline{3506}$ has 4 s.f. $0.000\underline{35600}$ has 5 s.f. $\underline{356}0$ has 3 s.f.

$\underline{3560}.$ has 4 s.f. $\underline{3.56} \times 10^{3}$ has 3 s.f . $\underline{3.560} \times 10^{3}$ has 4 s.f.

Significant figures and calculations What is the precision of the final answer when you perform calculations? Note that your calculations should be completed in full without any rounding until the final answer.

Multiplication and division
Pick the number in the initial question with the least number of significant figures. The answer has the same number of significant figures as this number.

Addition and subtraction
Pick the number with the least number of decimal places. The answer has the same number of decimal places as this number.

A combination of (1) and (2) Use the appropriate rule for the steps.

ALWAYS Use the rule of order in your calculations:
THAT IS – Brackets, Exponents, Multiplication/Division, Addition/Subtraction

WORKED EXAMPLES

(i) 36.4 ÷ **1.2** x 8.345 = **25**3.131667 (calc. answer) = 250 (**2 s.f**.) = 2.5×10^2

(ii) 17.54 – 1.**3** = 16.**2**4 (calc. answer) = 16.2 (**1 decimal place (d.p.)**)

(iii) 12.335 x **6.701** – 3.2 = 82.656835 – 3.**2** = 79.**4**56835 (calc. answer) = 79.5 (**1 d.p.**)

Exercise 3.

1. Specify the number of significant figures indicated in each of the following quantities or values.

(a) 307 metres (b) 26.98 kilojoules (c) 1.5200 (d) 0.001305 (e) 2 750 kilograms

(f) 20.060 litres (g) 2892000 to the nearest thousand people (h) 1.0×10^3

2. A grain of sand is weighed and found to have a mass of 650 mg. Write this mass

in scientific notation to (a) two significant figures

(b) three significant figures (c) four significant figures

3. An iceberg had a mass of 9,530kg. After three weeks floating in warm currents it has lost 64% of its mass. Find its remaining mass to

(a) the nearest kilo (b) the nearest hundred kilos (c) three significant figures

4. Perform the following calculations and give the answers in scientific notation to the correct number of significant figures:

(a) 5064 x 13 (b) 405.0 x 4.0 (c) 6.02 x 5.1 ÷ 0.00034

(d) 9.54 – 3.2 + 12.007 (e) 4.35 ÷ 9.1+1.7 (f) 12.8 + 9.08 x 7.1

ANSWERS TO EXERCISES

Conversion and orders of magnitude

Exercise 1

1.

Prefix	Symbol	In decimal notation	Order of magnitude
milli	m	0.001	-3
micro	μ	0.000001	-6
hecto	h	100	2
mega	M	1000000	6
deci	d	0.1	-1
centi	c	0.01	-2
kilo	k	1000	3
giga	G	1000000000	9

2. (a) 6000 metres (d) 1002000 micrograms (g) 725 grams
(b) 3200000 millilitres (e) 82.14 decilitres (h) 5.2 centimetres
(c) 0.044 metres (f) 0.12 kilograms (i) 40 micrograms

3. Larger by 1 order of magnitude. 4. (a) 3.5 x10^6 cm^3 (b) 0.00524 m^3

Scientific notation

Exercise 2.

1. (a)(i) 2.3 x10^2 (ii) 4.65 x10^4 (iii) 2 x10^{-2} (iv) 5.1 x10^{-3} (v) 5.85 x10^{12}
(b)(i) 6540 (ii) 431700 (iii) 0.015 (iv) 0.00255542
2. (a) 0.0142 m (b) 1.42 x10^{-2} m (c) 14.2 mm
3. (a) 7.68 x10^8 (b) 4.539 x10^2 (c) 1 x10^7 (d) 5.232 x10^{-1} (e) 2.31 x10^0
(f) 2.0 x 10^{-11} (g) 1.368 x10^7 (h) 5 x10^0 (i) 5.688 x10^5 (j) 8.239 x10^{10}
(k) 2 x10^2 (l) 8.15 x 10^{12}
4. 1.5539 x 10^7 km^2

Significant figures

Exercise 3.

1. (a) 3, (b) 4, (c) 5, (d) 4, (e) 3, (f) 5, (g) 4, (h) 2
2. (a) 6.5 x10^2, (b) 6.50 x10^2 , (c) 6.500 x 10^2
3. (a) 3431 kg, (b) 3400 kg, (c) 3430 kg
4. (a) 6.6 x 10^4 , (b) 1.6 x10^3, (c) 9.0 x 10^4, (d) 1.83 x10^1, (e) 2.2, (f) 7.7 x 10^1