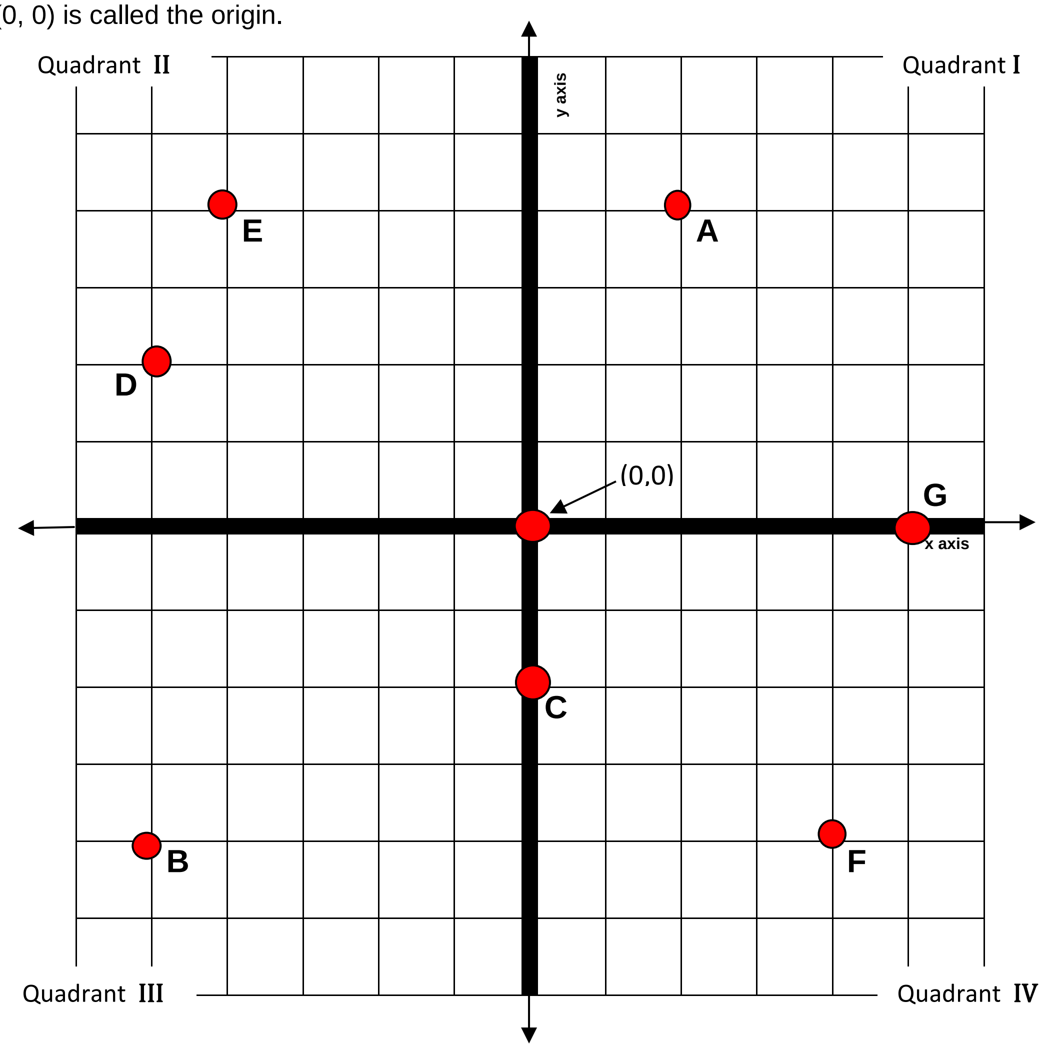
Module 7: Graphs

PLOTTING POINTS

An ordered pair of numbers (x,y) can be represented graphically as a point on 'xy plane' or the Cartesian plane. The value of the 'x coordinate' gives the position of the point in the horizontal direction. The value of the 'y coordinate' gives the position of the point in the vertical direction. The point (0, 0) is called the origin.



The Cartesian plane is divided into four quadrants.

I. Quadrant I, x and y both positive
II. Quadrant II, x is negative and y is positive
III. Quadrant III, x and y both negative
IV. Quadrant IV, x is positive and y is negative

Exercise 1

On the graph provided above,

a. Label the x axis and the y-axis from -5 to 5. Each grid line represents 1 unit.
b. Label the coordinate points for **A, B, C, D, E, F and G**?
c. Of the points you labelled in question 3, which point has a negative x -coordinate and a negative y-coordinate? Which point has a y-coordinate that is equal to zero?

DISTANCE BETWEEN POINTS

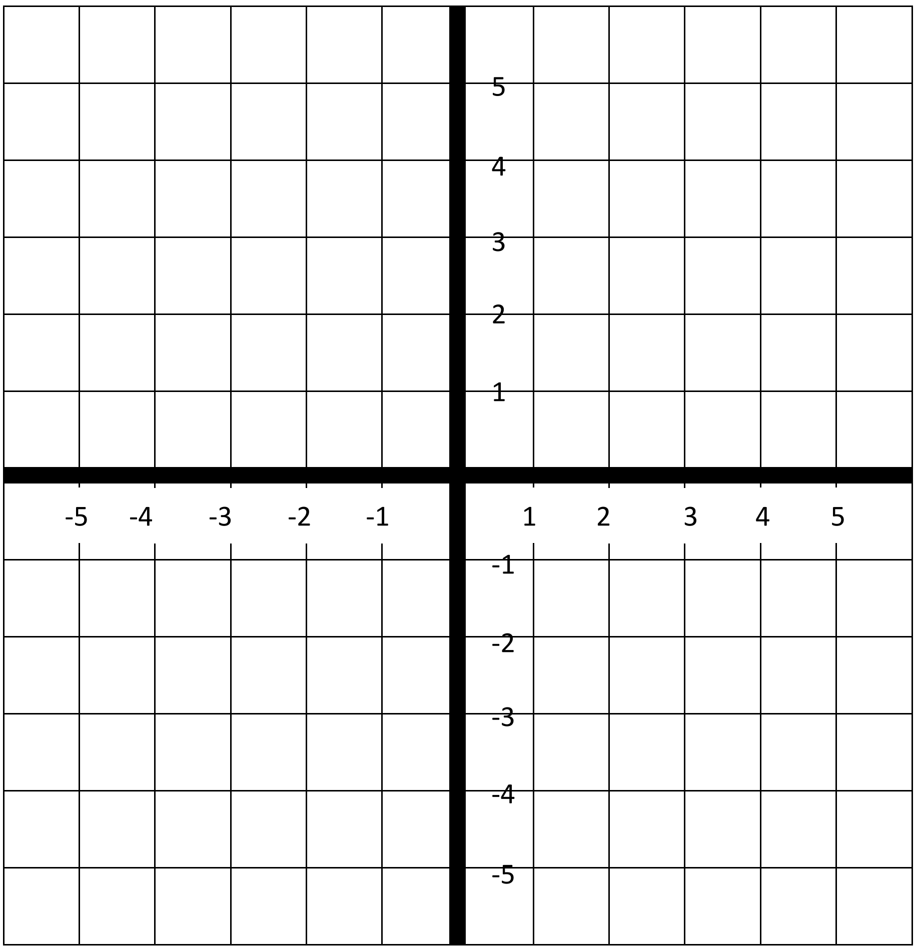
The distance between the two points (x_1,y_1) and (x_2,y_2), is given by the formula;

$$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}.$$

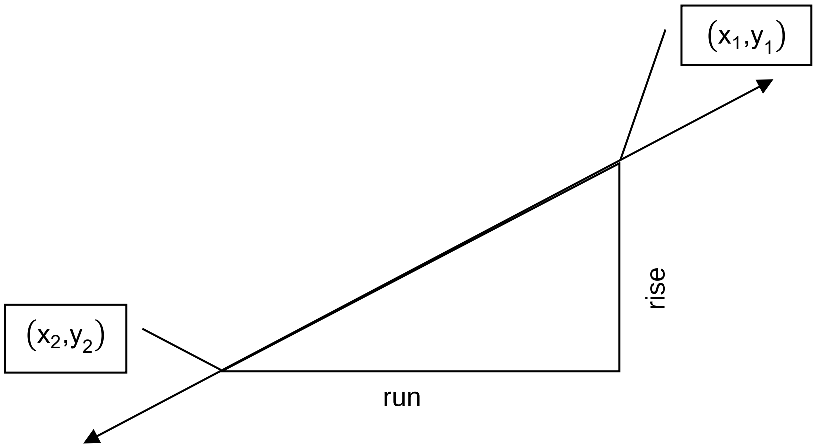
Exercise 2

Calculate the distance between two given points.

a. Plot the points (2,5) and (6,3) on the graph provided below and join the points with a line
b. We will let $(x_1 = 2, y_1 = 5)$ and $(x_2 = 6, y_2 = 3)$. Calculate $(x_1 - x_2)$, 'the difference in the x-coordinates', and $(y_1 - y_2)$, 'the difference in the y-coordinates'.
   $(x_1 - x_2) = (2 - 6) =$ $\qquad$ $(y_1 - y_2) = (5 - 3) =$
c. Now 'square the differences' calculated in part (b), that is find $(x_1 - x_2)^2 =$ $\qquad$ and $(y_1 - y_2)^2 =$
d. Add the two values calculated in part (c), that is find $(x_1 - x_2)^2+(y_1 - y_2)^2 =$
e. Last step. Take the square root of the value calculated in part (d). $\qquad$ $d = \sqrt{(x_1 - x_2)^2+(y_1 - y_2)^2} =$
f. Using the distance formula, calculate the distance between the two points (-4,4) and (5,0).



LINEAR EQUATIONS



A line is made by connecting two points; any two points can determine a line. The **slope (or gradient)** of a line measures the rate of change as we move along the line. The gradient can be positive, negative or zero. In other words, up-hill, down-hill or flat. The formula for the gradient of a line is:

$$m = \frac{\text{rise}}{\text{run}} = \frac{(y_2 - y_1)}{(x_2 - x_1)}.$$

Exercise 3

Calculate the gradient of the line joining the points **A** = (2,4) and **B** = (-5,-4).

1. Let A=(x_1,y_1)=(2,4) and B=(x_2,y_2)=(-5,-4)

a. Rise = $(y_2 - y_1)$ =

b. Run = $(x_2 - x_1)$ =

c. Gradient = $\frac{rise}{run}$ =

2. Calculate the gradient of the line joining the points **E** = (-4,4) and **C** = (0,-2) from Exercise 1.

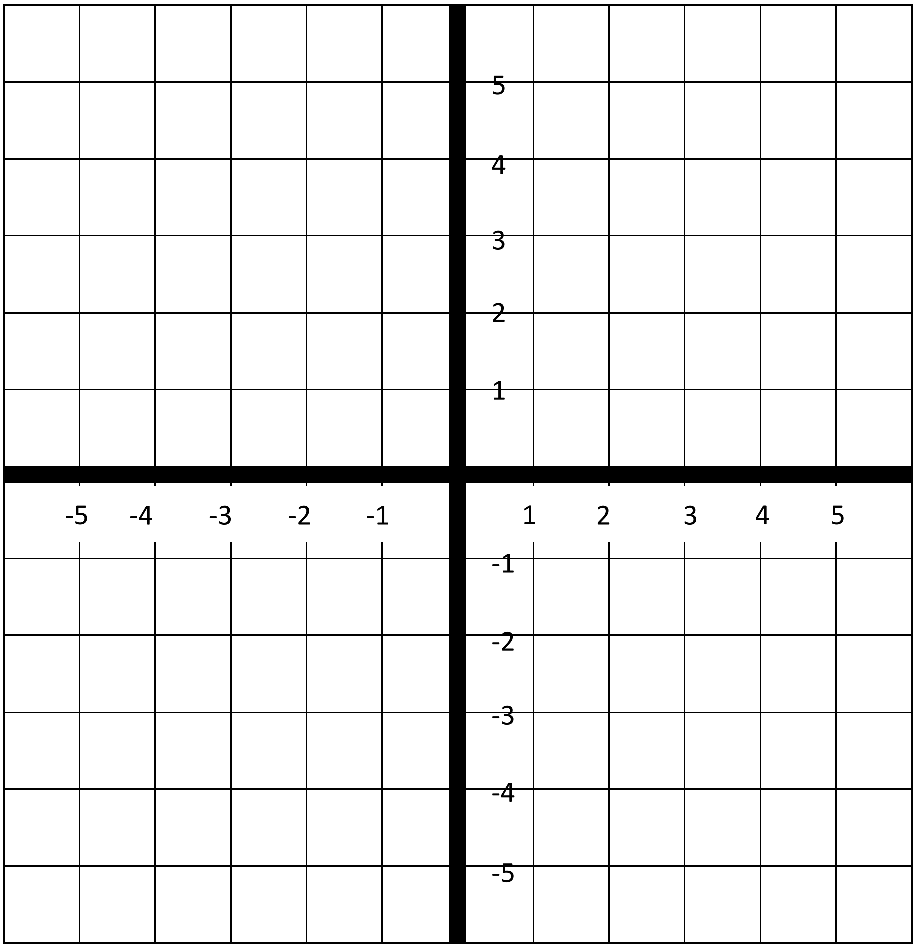
Graphing linear equations

It can be very useful to graph equations; we will look at the graphs of straight lines. Let's consider the equation y = 2x + 1. For each value of x we input into the equation, there will be a corresponding output, the y value. For example, when x = 1, y = 2(1)+1=3 and when x = -1, y = 2(-1)+1=-1. Each output value of y depends on the input value of x.

Using the linear equation, y = 2x + 1 , calculate the y - coordinate point for each of the x values given in the table below.

x	-3	-2	-1	0	1	2	3
y			-1		3		

Plot the calculated coordinate points on the graph below. All the points should fall in a straight line, join the points together and write the equation y=2x+1 along the line.



The value of the y-coordinate when x = 0 is called the **y-intercept.** This is where the graph of the line crosses the y-axis. The y-intercept for the above example, is 1 because when x=0, y=1.

The gradient of the line above is 2, for each unit of increase in x, the y value increases by 2, so the gradient must be 2. Check the gradient is 2 using *any* two points on the line.

Equation of a line

There are several ways to write the **equation of a line**, we will use the *slope-intercept* form,

$y = mx + c$ where m is the gradient of the line and c is the y-intercept.

e.g. The equation of the line which joins the points **E** and **C** is $y = -1.5x – 2$ as $m = -1.5$ and $c = -2$. (We calculated the gradient in Exercise 3 (q2), and from the graph, the y- intercept is -2.)

The equation of the line joining the points **A** and **B** is $y=\frac{8}{7}x + \frac{12}{7}$, we calculated the gradient in Exercise 3 (q1), but the y-intercept is not easily read from the graph of the line. In this case *any* point on the line can be used to find the value of the y-intercept, a worked example follows.

Worked example: Find the equation of the line which joins two coordinate points (2,4) and (-3,-1).

a. Calculate the gradient. For this line $m = \frac{4-(-1)}{2-(-3)} = \frac{4+1}{2+3} = \frac{5}{5} = 1$

b. Using $y=mx+c$ substitute $m=1$ into the equation, $y=1x+c$

c. To calculate c, substitute either of the coordinate points, (2,4) or (-3,-1) into y=1x+c. Let's use (2,4), it tells us that when x=2, y=4

$$4=1x2+c$$

$$4=2+c$$

$$4-2=2-2+c \quad \text{subtract 2 from both sides}$$

$$2=c \text{ or } c=2$$

Substituting m=1 and c=2 into the equation y=mx+c, we get y=1x+2, this is the equation of the line passing through the points (2,4) and (-3,-1). The slope for this line is 1 and the y-intercept for this line is 2.

Exercise 4

1. Following the steps a – c, in the worked example above, find the equation of the line joining the points

(-2 , -2) and (1, 10).

INTERPRETING GRAPHS

A graph may be used to the study relationship between a pair of variables. The graph provides a visual link between the numbers and any relationship that may exist. If there is a relationship we may want to describe it using a formula.

Exercise 5

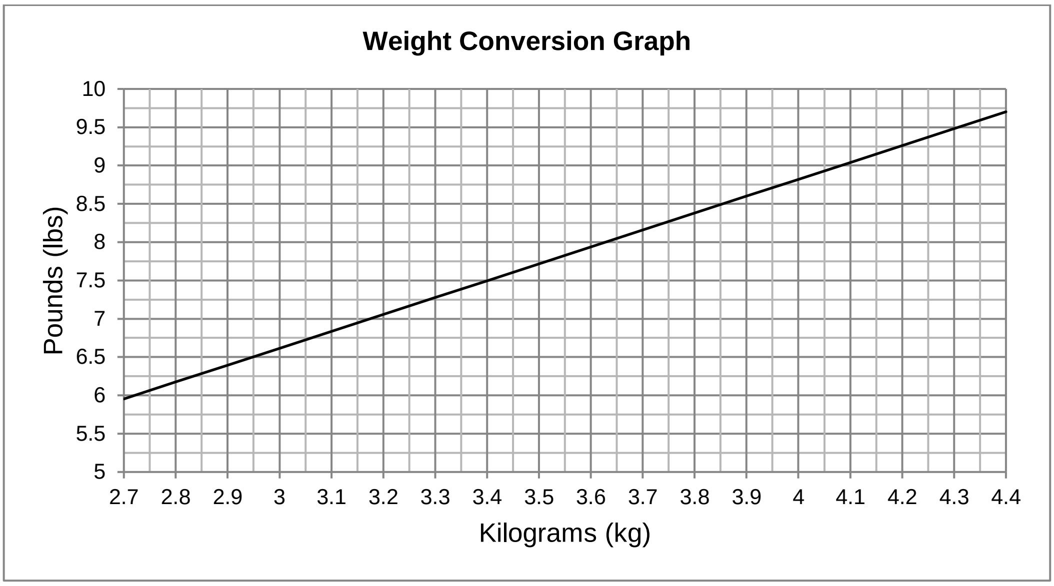
1. A family start at 10am to travel north from Perth to Geraldton, a distance of 425km. Between 12noon and 1pm they stop for lunch at a roadhouse 210km north of Perth. The family arrive in Geraldton at 3:30pm.

a. Roughly sketch a graph of the information given above, put time on the x axis and distance on the y axis.
The coordinate point for the start of the journey is (10.00,0), as the family sets out at 10am and has travelled 0 km. Using the 24 hour clock for time, figure out the three other coordinate points given in the question, plot them on the graph and connect the points with a line.

The gradient of the line will give us an estimate of the **average speed** of the car,

b. Calculate was the average speed of the car during the three time periods; 10.00-12.00, 12.00-13.00 and 13.00-15:30?

2.



a. Reading from the graph, convert 3.4kg to lbs. (Correct to 1 decimal place)
b. Reading from the graph, convert 9.5lbs to kg. (Correct to 1 decimal place)
c. What would be the ‘y-intercept’ of this graph given that 0kg must be 0 lbs?
d. Use the two points from parts a) and b) to calculate the gradient of the line?

e. Write down the linear equation for converting kilograms to pounds.

ANSWERS TO EXERCISES

PLOTTING POINTS

Exercise 1

b. A=(2,4) B=(-5,-4) C=(0,-2) D=(-5,2) E=(-4,4) F=(4,-4) G=(5,0)

c. Point in quadrant III: B. Point G has a zero y coordinate.

DISTANCE BETWEEN POINTS

Exercise 2

b. -4 2

c. $(-4)^2 = 16$ $2^2 = 4$

d. 16+4=20

e. $\sqrt{20} = 4.472$

f. $\sqrt{97} = 9.849$

LINEAR EQUATIONS

Exercise 3

a. -4 – 4 = -8

b. -5 – 2 = -7

c. Gradient = -8÷-7 ≈ 1.143

2. Gradient = -1.5

Exercise 4

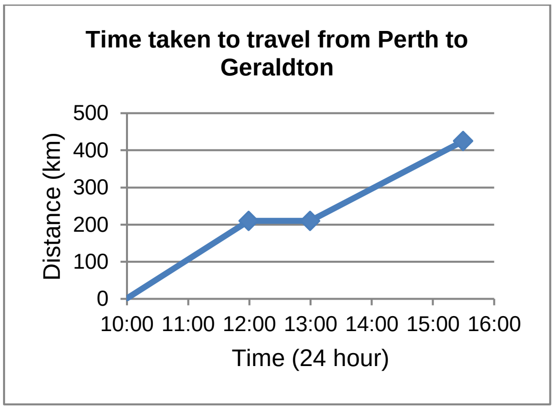
1. y = 4 x + 6 The line has a gradient of 4 and a y-intercept of 6.

INTERPRETING GRAPHS

Exercise 5

1.

a.



b.

105 km/hr, zero, 86 km/hr

2.

a. 7.5lbs

b. 4.3kg

c. zero

d. $2.2\frac{\text{lbs}}{\text{kg}}$

e. Pounds = 2.2 x kilograms